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Analysis of Thermal Buckling of Ceramic-Metal Functionally Graded Plates Using Refined Third Order Shear Deformation Theory

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Abstract
Functionally graded materials (FGMs) are microscopically inhomogeneous spatial composite materials, typically composed of a ceramic-metal or ceramic-polymer pair of materials. Therefore, it is important to investigate the behaviors of engineering structures such as beams and plates made from FGMs when they are subjected to thermal loads for appropriate design. Therefore, using an improved third order shear deformation theory (TSDT) based on more rigorous kinetics of displacements to predict the behaviors of functionally graded plates is expected to be more suitable than using other theories. In this paper, the improved TSDT is used to investigate thermal buckling of functionally graded plates.
Temperature dependent material property solutions are adopted to investigate thermal buckling results of functionally graded plates. To obtain the solutions, the Ritz method using polynomial and trigonometric functions for defining admissible displacements and rotations is applied to solve the governing equations.
Keywords— functionally graded materials, thermal loads, improved third order shear deformation theory, Ritz method

I. INTRODUCTION

Composite materials are known as the modern materials which are composed of two or more different materials, to have the desired properties in specified applications. The lightweight composite materials known as fiber-matrix laminated composites have been used successfully in aircraft, automotive, marine industries and other engineering applications. However, the mismatch in mechanical properties across the interface of two different materials may cause large inter-laminar stresses. Consequently, de-bonding and delimitation problems can occur, especially in a high temperature environment

To remedy such defects, functionally graded materials (FGMs), within which material properties vary continuously, have been proposed. The concept of FGM was proposed in 1984 by a group of materials scientists, in Sendai, Japan, for thermal barriers or heat shielding properties. FGM is one of the advanced high temperature materials capable of withstanding extreme temperature environments Typically, these materials are made from a mixture of ceramics and metal or a combination of different materials. The ceramic constituent of the material provides the high-temperature resistance due to its low thermal conductivity and protects the metal from oxidation. The ductile metal constituent, on the other hand, prevents fracture caused by stresses due to high-temperature gradient in a very short period of time.

The material transitions from a metal to a ceramic by increasing the percentage of ceramic material present in the metal until the appropriate percentage is reached or a pure ceramic is achieved (See Figure 1). [1]

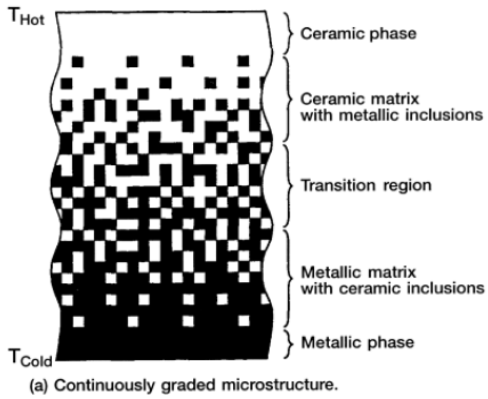

Figure 1: Graphical FGM representation of gradual transition in the direction of the temperature gradient.

Since the material does not have a dramatic change in material properties at any one point through the thickness, it would not cause a large stress concentration. This material usually exists where there is an extreme temperature gradient which is designated by T_{hot} and T_{cold} in Figure 1. The ceramic face of the material is generally exposed to a high temperature, while the metallic face is usually subjected to a relatively cooler temperature. The smooth transition of material properties allows for a material whose properties provide thermal protection as well as structural integrity reducing the possibilities of failure within the structure.

A. The Refined Plate Theory

An efficient and simple refined plate theory (RPT) was initially introduced and implemented by Shimpi and Patel (3) in order to deal with the problems of static and dynamic analysis of orthotropic plates. The refined theory can be classified among the third-order shear deformation theories. The development of the refined plate theory is based on the assumptions that the theory represents parabolic variations of shear strains (γ_{xz}, γ_{yz}) and shear stresses (σ_{xz}, σ_{yz}) throughout the plate thickness and also satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate. Additionally, the theory can provide high accuracy in prediction plate behavior subjected to mechanical loadings without using the shear correction factor.

Based on the basic assumptions of the RPT (Shimpi and Patel 3), the displacement field of the RPT can be written as follows:

$$u = u_0(x, y, t) - z w_{b,x} + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right) w_{s,x}$$

$$v = v_0(x, y, t) - z w_{b,y} + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right) w_{s,y}$$

$$w = w_b(x, y, t) + w_s(x, y, t).$$

The main differences between the improved TSDT developed by Shi (2) and the RPT are the middle terms of in-plane displacement functions. And the transverse displacement (w) of the RPT is composed of two components of the displacement due to bending (w_b) and shear (w_s).

To apply the improved theory for analyzing plate problems, it is begun with the Constitutive equations that take the form as,

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}, \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}$$

II. THE SOLUTION METHOD FOR FG PLATE ANALYSIS

The governing equation or the total energy functional based on the improved TSDT for FG plate analysis derived from the energy approach can be solved using the Ritz method in order to determine the thermal buckling results.

The total energy functional (Π) of FG plates for the thermal buckling analysis can be written as the following,

$$\Pi = U_e + V_e$$

Where U_e is strain energy and V_e potential energy due to thermal stress can be expressed as follows (5),

$$V_e = \frac{1}{2}\int_V [\sigma_{xx}{}^T d_{xx} + 2\sigma_{xy}{}^T d_{xy} + \sigma_{yy}{}^T d_{yy}]\, dV,$$

$$d_{ij} = u_{,i}u_{,j} + v_{,i}v_{,j} + w_{,i}w_{,j} \quad (i, j = x, y).$$

Two types of boundary conditions, which are Simply Supported (SS) and Clamped (CC), are chosen to consider the plate analysis.

For SS: the essential and natural conditions are:

$$u_0, w, \varphi_y, N_{xy}, M_{xx} = 0, (x = 0, a)$$

$$v_0, w, \varphi_x, N_{xy}, M_{yy} = 0, (y = 0, b)$$

The assumed in-plane displacements satisfying the conditions of SS in direction of x and y are

$$u_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} U_{mn}\cos\alpha_m x \sin\beta_n y$$

$$v_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} V_{mn}\sin\alpha_m x \cos\beta_n y$$

$$w_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}\sin\alpha_m x \sin\beta_n y$$

$$\varphi_x(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} \Phi_{xmn}\sin\alpha_m x \sin\beta_n y$$

$$\varphi_y(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} \Phi_{ymn}\sin\alpha_m x \sin\beta_n y$$

To consider the plate which is supported by clamped boundary condition denoted by CC, the conditions of this fully clamped are,

$$u_0, v_0, w, \varphi_x, \varphi_y, w_{,x} = 0, (x = 0, a)$$

$$u_0, v_0, w, \varphi_x, \varphi_y, w_{,x} = 0, (y = 0, b)$$

The assumed displacement and rotation functions that can satisfy the fully clamped conditions are expressed as following functions (4, 2):

$$u_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} U_{mn}\sin\alpha_m x \sin\beta_n y$$

$$v_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} V_{mn}\sin\alpha_m x \sin\beta_n y$$

$$w_0(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn}(1 - \cos 2\alpha_m x)(1 - \cos 2\beta_n y)$$

$$\varphi_x(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} \Phi_{xmn}\sin\alpha_m x \sin\beta_n y$$

$$\varphi_y(x, y) = \sum_{m=1}^{M}\sum_{n=1}^{N} \Phi_{ymn}\sin\alpha_m x \sin\beta_n y$$

Where $\alpha_m = \frac{m\pi}{a}, \beta_m = \frac{n\pi}{b}$ and $U_{mn}, V_{mn}, W_{mn}, \Phi_{xmn}, \Phi_{ymn}$ are unknown coefficients for thermal buckling of FG plates, the assumed displacement and rotation functions corresponding to boundary conditions are substituted into the total energy functional (Π) of Eq ($\Pi = U_e + V_e$) and then taking derivative the functional with respect to the unknown coefficients in the procedure of minimisation.

This procedure leads to a system of simultaneous equations equal in number to the number of unknown coefficient $U_{mn}, V_{mn}, W_{mn}, \Phi_{xmn}, \Phi_{ymn}$. The generalised eigenvalue problem for thermal buckling can be written as,

$$\left[[K] + \bar{\lambda}\,[K_T]\right][\Delta] = 0.$$

where $[K]$ and $[K_T]$ are the stiffness matrix and the coefficient matrix of temperature change, respectively, and the vector Δ is the eigenvector of the unknown coefficients. The parameter $\bar{\lambda}$ is the thermal buckling result which is equivalent to the critical temperature ΔT_{cr}. To calculate a set of thermal buckling, the determinant of the coefficient matrix in above Eq.is set to zero. In this paper the thermal buckling results of FG plates related to temperature dependent solution are also investigated by using the simple iterative technique.

III. APPLICATION OF THE IMPROVED TSDT TO FG PLATE ANALYSIS

An FG plate made of ceramic-metal according to the power law distribution is considered in this section. The geometry of FG plate is shown in Fig. 2, in which the material Composition at the top surface is assumed to be the ceramic-rich surface and the material Compositions are varied continuously to the metal-rich surface at the opposite side.

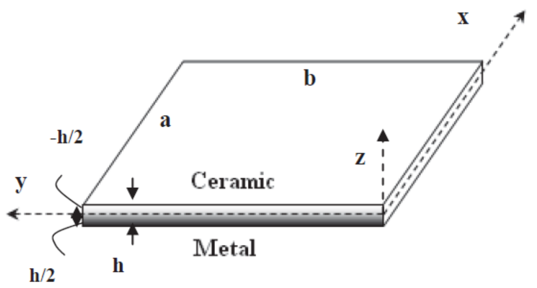

Figure 2: Thermal buckling of FG plates based on the improved TSDT

FG plate which consists of Al_2O_3 and steel supported by simply supported and clamped boundary conditions, are chosen to find out the results of critical buckling temperature. Temperature independent material property solution is implemented to obtain the analytical results.

(Critical Buckling Temperature of Simply Supported Al2O3/Steel plates (b/a=1.0) from temperature independent material properties)

TABLE I

Material	b/h=15	b/h=20	b/h=30	b/h=50	b/h=80
ceramic	781.39	445.344	199.826	72.293	28.287
n=0.2	637.901	363.653	163.201	59.048	23.106
n=1.0	499.958	285.039	127.928	46.288	18.113
n=2.0	459.696	262.198	117.714	42.599	16.67
n=5.0	423.153	241.465	108.443	39.251	15.361
metal	350.531	199.947	89.771	32.488	12.713

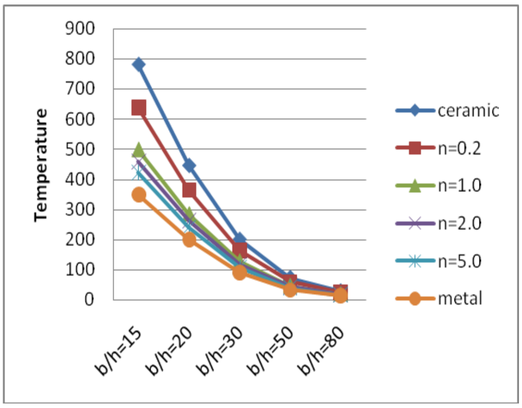

Figure 3: Effects of the plate thickness ratio on the critical buckling temperature of the simply supported Al2O3/Steel plates

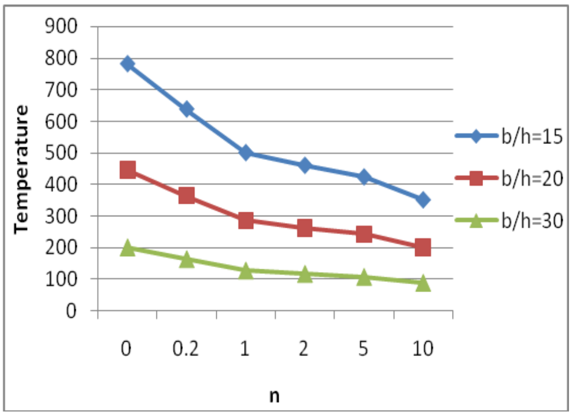

Figure 4: Critical bucking temperature versus n for Al2O3/Steel plate with simply supported boundary conditions

(Critical Buckling Temperature of Simply Supported Al2O3/Steel plates (b/a=1.0) from temperature dependent material properties)

TABLE II

Material	b/h=15	b/h=20	b/h=30	b/h=50	b/h=80
ceramic	1990.43	1170.79	548.973	219.171	103.634
n=0.2	1619.4	953.012	446.906	178.273	84.132
n=1.0	1255.03	738.585	346.28	138.023	65.035
n=2.0	1149.54	676.884	317.186	125.998	58.952
n=5.0	1061.94	625.766	293.108	116.03	53.888
metal	888.903	523.564	245.364	97.434	45.548

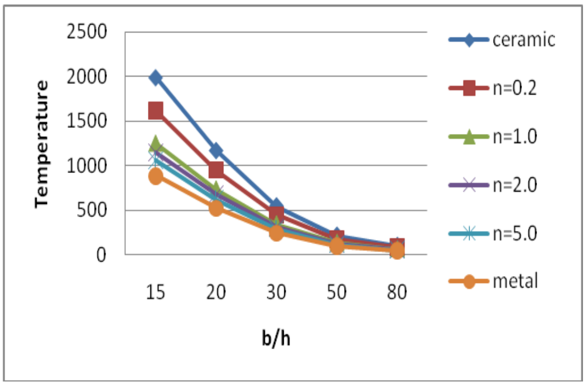

Fig 5: Effects of the plate thickness ratio on the critical buckling temperature of the Clamped Al2O3/Steel plates

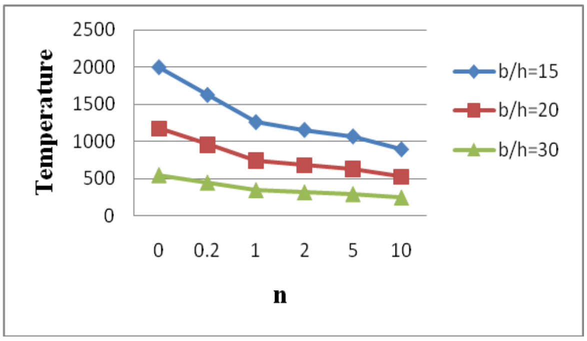

Fig 6: Critical bucking temperature versus n for Al2O3/Steel plate with Clamped boundary conditions

IV. DISCUSSION AND CONCLUSION:

1. The critical buckling temperatures of SS plates and CC plates made of Al2O3/Steel with several values of the volume fraction index are presented in (Table 1-2). The buckling temperature results in this table are derived from temperature independent solution with different thickness ratios.
2. Figures (3 and 5) demonstrates the influence of the thickness ratio (b/h) on the bucking temperature of a Simply supported and clamped FGM plate by increasing the thickness ratio (b/h) , the critical buckling temperature decreases.

Figure (4 and 6) demonstrates the influence of the power law index n on the bucking temperature of a Simply supported FGM plate . By increasing the power law index n, the critical buckling temperature decreases for k<1 rapidly and then slowly decreases for 1<k<10. It is noted that the buckling temperature of FGM beam decreases with the increase of the L/h ratio, as expected

Based on the investigated results, the conclusions of thermal buckling analysis of FG plates are presented as follows:

1. By using the improved TSDT for predicting thermal buckling results of FG plates,the theory reveals its significance when it is used to deal with thick FG plates (*b/h*≤20). This is due to the improved terms of the theory.
2. Increasing the value of the volume fraction index (*n*) leads to a reduction in the buckling temperature of FG plates made of Al2O3/Steel.

Based on the numerical results, this reveals that the highest buckling temperature is obtained from the Al2O3/Steel plate.

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